IDEA Public Schools

UNIT 6: EXPONENTIAL & LOGARITHMIC FUNCTIONS AND EQUATIONS
UNIT PLAN

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UNIT SYNOPSIS

In Algebra 1, students analyzed exponential functions and applied laws of exponents in mathematical and real-world problem situations. During this unit, students describe and analyze the inverse relationship between an exponential and logarithmic function, including the restrictions on domain and range, graph and write these inverse functions using notation such as $f^{-1}(x)$, and use composition of these functions, including domain restrictions, to determine if the functions are inverses of one another. Students graph the function $f(x) = \log_b(x)$ where b is 2, 10, and e, and analyze the key attributes such as domain, range, intercepts, and asymptotic behavior. Students determine the effects on the key attributes on the graph of $f(x) = \log_b(x)$ where b is 2, 10, and e when $f(x)$ is replaced by $af(x)$, $f(x) + d$, and $f(x - c)$ for specific positive and negative real values of a, c, and d, and investigate parameter changes and key attributes in terms of real-world problem situations. Students make connections between exponential and logarithmic equations by rewriting exponential equations as their corresponding logarithmic equations and logarithmic equations as their corresponding exponential equations. Students use this understanding to solve single logarithmic equations having real solutions and justify the reasonableness of the solutions. Students formulate exponential and logarithmic equations that model real-world situations, solve the equations, and determine the reasonableness of the solution in terms of the problem situation. In subsequent courses in mathematics, these concepts will continue to be applied to problem situations involving exponential and logarithmic functions and equations.

Modeling: Many real-world mathematical problems can be represented algebraically. These representations can lead to algebraic solutions. A function that models a real-world situation can then be used to make estimates or predictions about future occurrences.
- o   You can use repeated multiplication with a function in the form of $y = ab^x$ where b is a positive number other than 1.
- o   You can apply the four types of transformations – stretches, compressions, reflections, and translations – to exponential functions.
- o   Exponential functions with base e have the same properties as other exponential functions.
- o   If the numbers in a list follow a pattern, you may be able to use a rule to relate each number in the list to its numerical position.
- o   Logarithms and exponents have corresponding properties.

Equivalence: A single quantity may be represented by many different expressions.
The facts about a quantity may be expressed in many different equations.
- o   You can use logarithms to solve exponential equations, and you can use exponents to solve logarithmic equations.

Functions: A function is a relationship between variables in which each value of the input variable is associated with a unique value of the output variable. Functions can be represented in a variety of ways, such as graphs, tables, equations, or words. Each representation is particularly useful in certain situations. Some important families of functions are developed through transformations of the simplest form of the function.
- o   The exponential function $y = b^x$ is one-to-one, so its inverse $x = b^y$ is a function. To express "y as a function of x" for the inverse, write $y = \log_b(x)$.
- o   You can apply the four types of transformations – stretches, compressions, reflections, and translations – to logarithmic functions.
- o   Logarithmic functions with base e have the same properties as other logarithmic functions. The functions $y = e^x$ and $y = \log_e(x)$ are inverse functions. The relationship $a = e^b$ can be rewritten as $b = \log_e(a)$.

Misconceptions:

- Some students, when rewriting an exponential equation to its logarithmic equation form, may confuse the position of the argument and exponent. Students may rewrite the equation $y = b_x \to \log_b(x) = y$ rather than $y = b_x \to \log_b(y) = x$.
- Some students may mistakenly think that Common Log and Natural Log are equivalent or interchangeable in all scenarios.
- Some students may try to use Common Log on a calculator for logarithms of other bases without properly using the Change of Base Formula.

Underdeveloped Concepts:

- Students need to be able to use the properties of exponents to simplify exponential functions. These properties then have corollaries in logarithms.
- Students may not be familiar with compound interest formulas as former courses spend a lot of time on simple interest.
- Natural log can be used in any application problem

Key Questions:

- Exponential functions are characterized by a rate of change that is proportional to the value of the function and can be used to describe, model, and make predictions about problem situations.
- Logarithmic functions are characterized as inverses of exponential functions and can be used to describe, model, and make predictions about situations.
- How can key attributes be used to describe the behavior of exponential and logarithmic functions?
- How can the key attributes of exponential and logarithmic functions be used to make predictions and critical judgments?

CONTENT STANDARDS

Below are the standards addressed in this unit.

Readiness Standards	Supporting Standards
2A.2A Graph the functions $f(x) = \sqrt{x}$, $f(x) = 1/x$, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = |x|$, and $f(x) = \log_b(x)$ where b is 2, 10, and e, and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval.	**2A.2D** Use the composition of two functions, including the necessary restrictions on the domain, to determine if the functions are inverses of each other.
2A.2C Describe and analyze the relationship between a function and its inverse (~~quadratic and square root~~, logarithmic and exponential), including the restriction(s) on domain, which will restrict its range.	**2A.5B** Formulate exponential and logarithmic equations that model real-world situations, including exponential relationships written in recursive notation.
2A.5A Determine the effects on the key attributes on the graphs of $f(x) = b^x$ and $f(x) = \log_b(x)$ where b is 2, 10, and e when $f(x)$ is replaced by $af(x)$, $f(x) + d$, and $f(x - c)$ for specific positive and negative values of a, c, and d.	**2A.5C** Rewrite exponential equations as their corresponding logarithmic equations and logarithmic equations as their corresponding exponential equations.
2A.5D Solve exponential equations of the form $y = ab^x$ where a is a nonzero real number and b is greater than zero and not equal to one and single logarithmic equations having real solutions.	**2A.5E** Determine the reasonableness of a solution to a logarithmic equation.
2A.8C Predict and make decisions and critical judgments from a given set of data using linear, quadratic, and exponential models.	**2A.8A** Analyze data to select the appropriate model from ~~among linear, quadratic, and~~ exponential models.
	2A.8B Use regression methods available through technology to write an exponential function from a given set of data.

Focus on Disciplinary Literacy	Mathematical Process Standard (F) – Analyze mathematical relationships to connect and communicate mathematical ideas.
	Mathematical Process Standard (G) – Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

LEARNING SUPPORTS BY LESSON

There is a checkmark for the math support if the lesson	Math Supports	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13
makes a connection to prior content or from a previous unit or academic year	Access Prior Knowledge	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
uses familiar contexts or experiences to make the learning relevant to students	Real-World Connections	✓	✓	✓	✓	✓	✓	✓					✓	
makes use of graphic organizers	Graphic Organizers	✓	✓	✓		✓	✓	✓	✓					
includes tools like rulers, protractors, patty paper, algebra tiles, etc.	Tools or Manipulatives													
incorporates tables, reference charts, displays, pictures, models, or color-coding	Visual Aids			✓	✓		✓	✓	✓					
includes definitions, examples vs. nonexamples, cognates, etc.	Vocabulary Supports	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓		✓
includes strategies that support language development														
asks students to discuss with their partner to prepare for whole class discussion	- Turn and Talk	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓
teacher facilitates a whole class discussion to debrief key learnings	- Guided Discussion	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
asks students to think independently, test their idea with a partner, and share whole group	- Think, Pair, Share													
includes sentence stems to support students with explanations	- Sentence Stems									✓		✓		
provides opportunities for students to work with a partner or a group	Peer Collaboration	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
uses mnemonics such as SohCahToa	Mnemonics													
includes websites or equipment that enhances the lesson	Technological Support	✓	✓	✓	✓	✓	✓	✓	✓					
content can be presented in different forms														
uses hands-on tools or manipulatives to represent the math	- Concrete													
uses drawings to represent the math	- Pictorial	✓	✓	✓		✓	✓	✓	✓	✓	✓			
uses numbers and number sentences to represent the math	- Abstract	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

The EFFL Model

Here are helpful resources that you guide you in the right direction before your EFFL lesson!

Why Should We EFFL?

The article advocates for the Experience First, Formalize Later (EFFL) teaching model, emphasizing its effectiveness in fostering deep understanding and flexible thinking in students. The author compares traditional teaching to a game of "Simon Says," where students merely mimic instructions without grasping underlying concepts. In contrast, EFFL encourages students to engage actively with problems, enhancing their ability to understand and apply math concepts creatively.

Tips for Lesson Planning

The article offers practical advice for effective lesson planning beyond the exhaustive and overly detailed approaches often emphasized during teacher training. It underscores the importance of thoughtful preparation but rejects the notion that teachers need to script every minute or detail of a class session.

Making the Most of Your EFFL Lesson Debrief

The article discusses the significance of the debriefing phase in the Experience First, Formalize Later (EFFL) lesson model, emphasizing its role in reinforcing learning and highlighting student contributions. The debrief session is seen as crucial for integrating academic vocabulary, emphasizing key lesson understandings, and valuing students' mathematical insights.

While You EFFL!

While each lesson may be unique in context and skills, all lessons benefit from the following practices:

Teacher Look Fors:
- ❑ Utilizing the Do Now to spark students' interest in the Activity.
- ❑ Use questioning to promote small group discussion and exploration, guided by monitoring questions.
- ❑ Connects Experience First to formal concepts using a colored pencil/pen to take notes along the margin during the Debrief.
- ❑ Facilitates whole-class discussions for students to reflect, share insights, and provides feedback that reinforces key concepts.
- ❑ Tracks time to adapt lesson pacing and support based on student response and engagement.

Students Look Fors:
- ❑ In the Activity, students engage in group work and discourse.
- ❑ Exploring the activity, testing hypotheses and approaches (trial & error).
- ❑ Take notes on key ideas and concepts using different colored pencil/pen to take notes along the margin.
- ❑ Share thoughts and ideas that demonstrate their approach to their work.

Other considerations
- During the **Experience First** phase, if most of your students seem stuck or disengaged, take a moment to pause, reset, and provide clear instructions. Some problems of the Activity are more suitable to do a whole-class discussion as a means to save some instructional time for Student Practice or the Exit Ticket. You are encouraged to adapt the EFFL (Experience First, Formalize Later) process to meet your students' needs while maintaining a focus on student-centered instruction.

ROADMAP

Day	Date	Lesson	Lesson Title
AT A GLANCE: Unit 6 – Exponential & Logarithmic Functions and Equations			
Day	Date	Lesson	Lesson Title
1		1	Investigating Exponential Rates of Change
2		2	Writing and Describing Exponential Functions
3		3	Graphs of Exponential Functions
4		4	Transformations of Natural Base Exponential Functions
5		5	Applications of Exponential Functions
6			*Unit 6 Success Day 1 –Review and Reteach topics from 6.1 - 6.5*
7		6	Building Exponential Models
8		7	Logarithms
9		8	Graphs of Logarithmic Functions
10			*Unit 6 Success Day 2 –Review and Reteach topics from 6.6 - 6.8*
11		9	Developing the Product, Quotient, and Power Properties of Logarithms
12		10	Inverses of Exponentials & Logs and Inverse Properties of Logs
13		11	Applying Logarithm Properties in Any Base & Change of Base Formula
14			*Unit 6 Success Day 3 –Review and Reteach topics from 6.9 - 6.11*
15		12	Solving Exponential Equations (Using Logs)
16		13	Solving Logarithmic Equations
17			*Unit 6 Success Day 4 – Unit Assessment Review*
18			End of Unit 6 Assessment

Date: _____
Lesson 1: Investigating Exponential Rates of Change

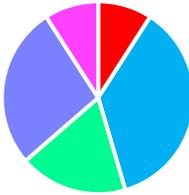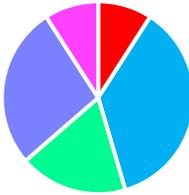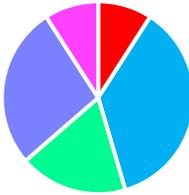| Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors
◆ **2A.8A** Analyze data to select the appropriate model from among linear, ~~quadratic~~, and exponential models.	**Necessary Materials and Pre-Lesson Prep** ▪ SE workbook ▪ graphing calculator Lesson Structure: ■ Do Now (7 min) ■ INM (20 min) ■ Debrief (4 min) ■ Student Practice (25 min) ■ Exit Ticket (4 min) **Mathematical Goal of this Lesson** In this lesson, students will investigate and describe rates of change for linear and exponential data. The lesson starts with an exploratory activity of how the data can be represented in tables or graphs. **Opportunities to CFU** ✓ INM #2-7 ✓ Student Practice #1-2 ✓ Debrief **Other Notes to Inform Your Planning** This lesson introduces exponential functions through investigating rates of change that students are familiar with from earlier units and Algebra 1. This standard is continued in the next lesson. **Focus on Disciplinary Literacy** INM & Student Practice The students need to practice diverse types of problems and select the correct type of relationship for the data presented. Later in this unit, students will have the opportunity to use Desmos to graph the data and confirm that their predictions were appropriate for the data set, this will allow students to develop their understanding and eliminate common misconceptions.	**Look for teachers to…** ❑ Guide students through comparing the change in y to determine if the relation is a linear or exponential function. ❑ Focus students on the difference in the properties of linear and exponential functions. **Look for students to…** ❑ Determine if a relationship is linear or exponential. ❑ Use the y-values in a table of values to find the difference and the ratios of subsequent terms. **Student Know/Do Chart** Students can **Do** Use the change in outputs of a relation or function to determine if there is a common difference or a common ratio. **Know** Linear expression varies at a constant rate (additively) while an exponential relation varies multiplicatively (ratio).

Important Vocabulary
- linear
- nonlinear
- rate of change
- constant
- ratio

Date: _____
Lesson 2: Writing and Describing Exponential Functions

Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors
◆ **2A.8A** Analyze data to select the appropriate model from among linear, ~~quadratic~~, and exponential models. ◆ **2A.5B** Formulate exponential ~~and logarithmic~~ equations that model real-world situations, including exponential relationships written in recursive notation.	<u>Necessary Materials and Pre-Lesson Prep</u> ▪ SE workbook ▪ graphing calculator **Lesson Structure:** 🟥 Do Now (7 min) 🟦 INM (25 min) 🟩 Debrief (4 min) 🟪 Student Practice (20 min) 🟪 Exit Ticket (4 min) <u>Mathematical Goal of this Lesson</u> In this lesson, students will write exponential functions and describe the growth/decay factor. The lesson starts with an explanation of the exponential functions and the characteristics of its equation. Students will answer conceptual questions about exponential functions and will be able to predict outcomes depending on the models provided.	<u>Look for teachers to...</u> ❑ Focus students on the kinds of mathematical and real-world situations that exponential functions can model. ❑ Have students consider the growth or decay factor in relation to the given scenario. <u>Look for students to...</u> ❑ Set up models using the common ratio to make predictions for exponential growth or decay.
Important Vocabulary ▪ linear ▪ nonlinear ▪ rate of change ▪ constant ▪ ratio ▪ **common ratio** ▪ **exponential growth** ▪ **exponential decay** ▪ **growth factor** ▪ **decay factor**	<u>Opportunities to CFU</u> ✓ INM #4-10 ✓ Student Practice #1-5 ✓ Debrief <u>Other Notes to Inform Your Planning</u> This lesson builds on the prior lesson and focuses on if there is growth or decay in the scenario. Students will write functions using the standard form of an exponential function and will use the rate of change to determine if it is growing or decaying exponentially. The students will practice different types of problems and questioning, that lead to a better understanding of the exponential functions, their behavior and applications to real-world situations. **Focus on Disciplinary Literacy** INM	**Student Know/Do Chart** Students can **Do** Use exponetial growth or decay models to solve real word problems. **Know** The initial value is the value of the function that is obtained by having an exponent of zero. **Know** For Exponential Functions $f(x) = a \cdot b^x$, a is the initial value, b is the common ratio or multiplier (a and b are constant), x is the input data, and $f(x)$ is the output of the function.

Date: _____
Lesson 3: Graphs of Exponential Functions

Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors

Standard(s)

◆ **2A.2A** Graph the functions $f(x) = \sqrt{x}$, $f(x) = 1/x$, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = |x|$, and $f(x) = \log_b(x)$ where b is 2, 10, and e, and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval.

◆ **2A.5A** Determine the effects on the key attributes on the graphs of $f(x) = b^x$ and $f(x) = \log_b(x)$ where b is 2, 10, and e when $f(x)$ is replaced by $af(x)$, $f(x) + d$, and $f(x - c)$ for specific positive and negative values of a, c, and d.

Important Vocabulary
- domain
- range
- **asymptotic behavior**
- **horizontal asymptote**
- y-intercept

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep
- SE workbook
- graphing calculator
- laptops for Desmos

Lesson Structure:
- 🟥 Do Now (5 min)
- 🟦 INM (20 min)
- 🟩 Debrief (10 min)
- 🟪 Student Practice (15 min)
- 🟪 Exit Ticket (5 min)
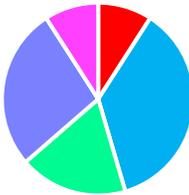
Mathematical Goal of this Lesson
In this lesson, students will learn how to graph exponential functions and determine the domain, range, y-intercept, and asymptote. Then identify if it is modeling growth or decay. Students will write equations for transformed exponential equations with given features.

Opportunities to CFU
- ✓ INM problem #2-5
- ✓ Debrief
- ✓ Student Practice #1-5

Other Notes to Inform Your Planning
Exponential functions are a little different from the other families of functions we have explored because there isn't just one parent function. Instead, we are looking at some examples of parent functions that are of the form $y = b^x$.
Students will need to use Desmos and 3 different colored writing utensils (not their red pens) to investigate the graphs of the exponential functions.

Focus on Disciplinary Literacy

INM & Debrief

Lesson Look Fors

Look for teachers to…
- ❑ Utilize Nspires or Desmos to support students in analyzing exponential function graphs.
- ❑ Focus students on what stays the same and what changes for different exponential curves with the different equations.

Look for students to…
- ❑ Graph a variety of exponential functions.
- ❑ Analyze and describe the key features of an exponential graph and connect the equation to the intercepts and asymptote.

Student Know/Do Chart

Students can
- **Do** Describe the domain and range of an exponential graph.
- **Do** Determine the horizontal asymptote in an exponential function.
- **Know** In exponential functions, changes of $\pm h$ in the exponent will cause a horizontal shift and $\pm k$ after the exponential term will cause a vertical shift.
- **Know** Growth and decay can be determined by the value of the base, b. $b > 1$ is growth and $0 < b < 1$ is decay.

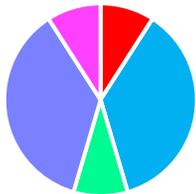Date: _____
Lesson 4: Transformations of the Natural Base Exponential Functions

Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors

Standard(s)

◆ **2A.5A** Determine the effects on the key attributes on the graphs of $f(x) = b^x$ and ~~$f(x) = \log_b(x)$~~ where b is ~~2, 10, and~~ e when $f(x)$ is replaced by $af(x)$, $f(x) + d$, and $f(x - c)$ for specific positive and negative values of a, c, and d.

Important Vocabulary
- asymptotic behavior
- transformation
- translation
- reflection
- compressions

Notes for Intellectual Preparation & Lesson Planning

<u>Necessary Materials and Pre-Lesson Prep</u>
- SE workbook
- graphing calculator
- GeoGebra Applet
- Desmos

Lesson Structure:
- Do Now (6 min)
- INM (25 min)
- Debrief (4 min)
- Student Practice (20 min)
- Exit Ticket (5 min)
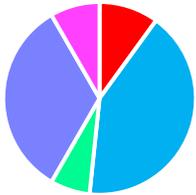
<u>Mathematical Goal of this Lesson</u>
In this lesson, students will describe and graph transformations of natural base exponential functions. Students will be introduced to the number e and the characteristics of the graph. Then, students will use their understanding of transformation of graphs and will apply the rules to exponential functions. Students will answer conceptual questions about exponential functions and their respective transformation.

<u>Opportunities to CFU</u>
- ✓ INM problems #2-6
- ✓ Debrief
- ✓ Student Practice #1-8

<u>Other Notes to Inform Your Planning</u>
This lesson builds on the prior lesson and focuses on the transformations of exponential functions with base of e, the natural number. The prior lesson focused on the same transformations, but now with e as the base.

Focus on Disciplinary Literacy
INM & Student Practice

Lesson Look Fors

<u>Look for teachers to…</u>
- ❑ Connect today's lesson to the prior lesson but focus on the natural number, e.
- ❑ Focus students on how the base effects the key features of an exponential function.

<u>Look for students to…</u>
- ❑ Interpret the changes to the graph of a family of base e exponential functions caused by changing a, k, c, & d.
- ❑ Sketch the base e exponential function curves and label the key features.

Student Know/Do Chart

Students can

Do Describe the key features of the base e exponential function graph and transformations caused by given values.

Know A vertical shift is represented by $f(x) = ae^x + c$ or $f(x) = ae^x - c$.

Know A horizontal shift is represented by $f(x) = ae^{x-d}$ or $f(x) = ae^{x+d}$.
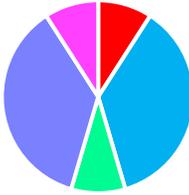
Date: _____
Lesson 5: Applications of Exponential Functions

Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors

Standard(s)

◆ **2A.5B** Formulate exponential ~~and logarithmic~~ equations that model real-world situations, including exponential relationships written in recursive notation.

Important Vocabulary
- compound interest
- compound daily
- compound quarterly
- compound monthly
- compound yearly
- compounding
- principal
- interest rate
- balance

Notes for Intellectual Preparation & Lesson Planning

<u>Necessary Materials and Pre-Lesson Prep</u>
- SE workbook
- laptops for Desmos or graphing calculator

Lesson Structure:
- 🟥 Do Now (5 min)
- 🟦 INM (20 min)
- 🟩 Debrief (10 min)
- 🟪 Student Practice (15 min)
- 🟪 Exit Ticket (5 min)

<u>Mathematical Goal of this Lesson</u>
In this lesson, students will write exponential functions for different growth and decay using a percent rate of change. Students will create and apply a formula for compound interest.

<u>Opportunities to CFU</u>
- ✓ After INM part I and part II
- ✓ Debrief
- ✓ Student Practice #1-6

<u>Other Notes to Inform Your Planning</u>
One of the most common examples used for exponential growth is interest rate. There are different kinds of interest models. Our goal today is for students to create a formula for compound interest. Before teaching this lesson, you will need to think about what prior knowledge your students have.
Depending on where your students are at, you may need to spend a little time on the front end of this lesson talking through some of these ideas.

Lesson Look Fors

Look for teachers to...
- ❑ Support students on understanding the various terms in context of exponential growth and decay with money.
- ❑ Depending on data and your students' experience you may need more or less baseline support for students in this lesson.

Look for students to...
- ❑ Solve a variety of problems in context using compound interest.
- ❑ Compare how different rates and number of times compounded will affect the balance of an account.

Student Know/Do Chart

Students can
- **Do** 🎯 Write exponential functions for different growth and decay using a percent rate of change.

- **Know** 🎓 Compound interest can be used as an application of exponential growth or decay using the formula $Balance = P(1 + \frac{r}{n})^{nt}$. Where $P =$ principal, $r =$ interest rate, $n =$ number of times compounded, and $t =$ time (usual in years).
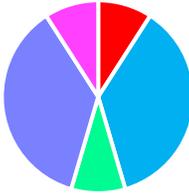
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Lesson 6: Building Exponential Models

Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors

Standard(s)

- ◆ **2A.8C** Predict and make decisions and critical judgments from a given set of data using ~~linear, quadratic, and~~ exponential models.
- ◆ **2A.8B** Use regression methods available through technology to write an exponential function from a given set of data.

Important Vocabulary

- **exponential regression**
- **expreg**
- **r²**
- growth rate
- y-intercept

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep
- SE workbook
- graphing calculator
- laptop for Desmos
- racquetball or tennis ball per group
- meter stick per group

Lesson Structure:
- 🟥 Do Now (5 min)
- 🟦 INM (20 min)
- 🟩 Debrief (10 min)
- 🟪 Student Practice (15 min)
- 🟪 Exit Ticket (5 min)
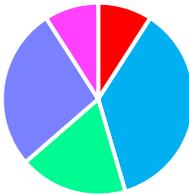
Mathematical Goal of this Lesson
In this lesson, students will create exponential models to fit a data set using the y-intercept and an approximated percent rate of change or with regression using technology. Also, students will write an exponential model to fit two given points.

Opportunities to CFU
- ✓ INM problem #2-4
- ✓ Debrief
- ✓ Student Practice #1-2

Other Notes to Inform Your Planning
You will need 1 racquetball and 1 meter stick or measuring tape for each group. We're going to be collecting data on the height of each bounce when you drop a racquetball and then writing an exponential model for the data. In addition to a ball and measuring stick, groups will also need a device to record a video of the bounces. Slow motion videos work best for measuring the height of the bounce. One student should hold the meter stick while another student drops the racquetball from 100 cm. One of the students can then take the video so that they can see each bounce and meter stick in view so they can use the meter stick for measurement. You'll want to have students work quickly with the data collection, so you have time to get through everything. Five minutes should be enough time to get everything they need and to complete the table in question #1.

Focus on Disciplinary Literacy

INM & Debrief

Lesson Look Fors

Look for teachers to...
- ❑ Support students in collecting real data for their group.
- ❑ Support students to write an exponential equation from two points after exploring their graphs.

Look for students to...
- ❑ Work in groups to conduct a bouncing ball experiment and to collect real data.
- ❑ Identify the y-intercept of their data and what it means in context, then calculate a common ratio based on the data.

Student Know/Do Chart

Students can
- 🎯 **Do** Write an exponential model to fit two given points.
- 🎯 **Do** Use real-world data to create a graph and approximate an exponential relationship.
- 🎓 **Know** If two points are consecutive, to find the common ratio between them, divide the y-values. If two points are not consecutive, find out how many times r gets multiplied, write an equation and solve for r.
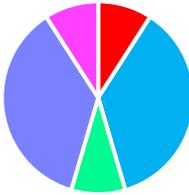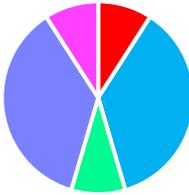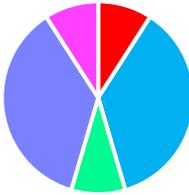
Date: _____
Lesson 7: Logarithms

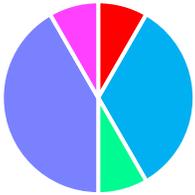| Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors
◆ **2A.2C** Describe and analyze the relationship between a function and its inverse (quadratic and square root, logarithmic and exponential), including the restriction(s) on domain, which will restrict its range. ◆ **2A.5C** Rewrite exponential equations as their corresponding logarithmic equations and logarithmic equations as their corresponding exponential equations.	<u>Necessary Materials and Pre-Lesson Prep</u> ▪ SE workbook ▪ graphing calculator or Desmos **Lesson Structure:** 🟥 Do Now (5 min) 🟦 INM (20 min) 🟩 Debrief (10 min) 🟪 Student Practice (15 min) 🟪 Exit Ticket (5 min) <u>Mathematical Goal of this Lesson</u> In this lesson, students will rewrite exponential equations as logarithmic equations and vice versa. Students will evaluate logarithmic expressions and use them to solve equations. <u>Opportunities to CFU</u> ✓ After the INM ✓ Student Practice #1-3 ✓ Debrief <u>Other Notes to Inform Your Planning</u> The mission of the lesson is to try to find a way to help students understand logarithms better than they would if they were just introduced to them and told how to plug-in values and evaluate them. We want students to see the relationship as an inverse of exponential functions and to understand how the base, exponent, and output of an exponential equation are represented in the corresponding logarithmic equation.	<u>Look for teachers to...</u> ❑ Get students to consider the logarithmic function for what it really does before introducing "log" by first calling it the "IDEA" function. ❑ Use the margin notes time as an opportunity to formalize what is a logarithm and how we use the notation. <u>Look for students to...</u> ❑ Learn to evaluate a log expression using the connected exponential expression. ❑ Look for patterns to develop an understanding of how log expressions work as inverses operations of exponential expressions.
Important Vocabulary		**Student Know/Do Chart**
▪ **logarithm** ▪ base, b ▪ **common logarithmic function, $y = \log(x)$** ▪ **logarithmic function, $f(x) = \log_b x$** ▪ **natural logarithmic function, $y = \ln x$**		Students can **Do** 🎯 Rewrite exponential equations as logarithmic equations and vice versa. **Know** 🎓 Logarithmic function is the inverse of an exponential function. $b^x = y \leftrightarrow \log_b y = x.$ **Know** 🎓 $\log_b b^x = x$ and $b^{\log_b x} = x$

Date: _____
Lesson 8: Graphs of Logarithmic Functions

Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors
◆ **2A.2A** Graph the functions $f(x) = \sqrt{x}, f(x) = 1/x,$ $f(x) = x^3, f(x) = \sqrt[3]{x}, f(x) =$ $b^x, f(x) = |x|,$ and $f(x) = \log_b(x)$ where b is 2, 10, and e, and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval. ◆ **2A.2C** Describe and analyze the relationship between a function and its inverse (quadratic and square root, logarithmic and exponential), including the restriction(s) on domain, which will restrict its range.	<u>Necessary Materials and Pre-Lesson Prep</u> ▪ SE workbook ▪ graphing calculator or Desmos Lesson Structure: ■ Do Now (5 min) ■ INM (15 min) ■ Debrief (10 min) ■ Student Practice (20 min) ■ Exit Ticket (5 min) <u>Mathematical Goal of this Lesson</u> In this lesson students will graph logarithmic functions and their transformations and analyze key attributes, such as domain, range, intercepts, end behavior, and asymptotes. Students will also describe a logarithmic function as the inverse of an exponential function. <u>Opportunities to CFU</u> ✓ INM #2-6 ✓ Student Practice #1-5 ✓ Debrief <u>Other Notes to Inform Your Planning</u> In lesson 6.3, students learned about the graph of an exponential function including the y-intercept, asymptote, domain and range. Today we will be doing the same with the graph of a logarithmic function. In addition, we want to connect how the graphs of the exponential and log functions are related because they are inverse functions. We are going to connect a few different points of prior learning. You'll notice in the activity that we start by first working with the exponential function, $f(x)$, and then will investigate the logarithmic function, $g(x)$.	<u>Look for teachers to…</u> ❑ Formalize for students that exponential and log functions are inverses. ❑ Help students focus on the key features of log functions and how those relate to what we learned in lesson 6.3 about exponential functions. <u>Look for students to…</u> ❑ Create tables of values to graph a log function and compare it to its inverse. ❑ Interpret the meaning of logarithmic expressions in terms of exponential equations.
Important Vocabulary ▪ domain ▪ range ▪ end behavior ▪ horizontal asymptote ▪ vertical asymptote ▪ x-intercept ▪ y-intercept ▪ transformation		**Student Know/Do Chart** Students can **Do** Graph logarithms with base 2, 10, or e and analyze the key features of the graph. **Know** Logarithms are functions that will undo an exponent. Logarithms are the inverses of exponential functions. **Know** The logarithmic parent function has x-intercept: (1,0), asymptote: $x = 0$, domain: $\mathbb{R} > 0$, and range: $\mathbb{R}$.

Date: _____
Lesson 9: Developing the Product, Quotient, and Power Properties of Logarithms

Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors
◆ **2A.5D** Solve exponential equations of the form $y = ab^x$ where a is a nonzero real number and b is greater than zero and not equal to one and ~~single~~ logarithmic equations having real solutions.	**Necessary Materials and Pre-Lesson Prep** ▪ SE workbook **Lesson Structure:** 🟥 Do Now (5 min) 🟦 INM (20 min) 🟩 Debrief (5 min) 🟪 Student Practice (20 min) 🟪 Exit Ticket (5 min) **Mathematical Goal of this Lesson** In this lesson, students will develop and apply the Product Property, Quotient Property, and Power Property of Logarithms. The exponential product, quotient, and power properties should be reviewed and connected to what that will mean in logarithms, what happens to the powers happens to the logs.	Look for teachers to... ❑ Focus students on logarithms having the same base before they can be combined. The base is not affected when combined. Look for students to... ❑ Combine and separate logarithms of the same base using product, quotient, and power properties of logarithms. ❑ Use the definition of logarithms to simplify logarithmic expressions.
Important Vocabulary ▪ argument ▪ logarithm ▪ base ▪ product property of logarithms ▪ quotient property of logarithms ▪ power property of logarithms	**Opportunities to CFU** ✓ INM #1, 4, & 7 ✓ Student Practice #1-6 ✓ Debrief **Other Notes to Inform Your Planning** The properties of exponential operations have been well established since Algebra 1 and they are a good base to build upon in this lesson while connecting to logarithms as the inverse of exponentials. Be sure students are able to use the product, quotient, and power properties in both directions to be ready to combine or separate logarithms in order to solve equations later in the unit.	**Student Know/Do Chart** Students can 🎯 **Do** Rewrite logarithmic expressions using the product, quotient, and power properties. 🎓 **Know** The Product Property of Logarithms: $\log_b m + \log_b n = \log_b mn$ 🎓 **Know** The Quotient Property of Logarithms: $\log_b m - \log_b n = \log_b \frac{m}{n}$ 🎓 **Know** The Power Property of Logarithms: $\log_b m^n = n \log_b m$

Date: _____
Lesson 10: Inverses of Exponentials & Logs and Inverse Properties of Logs

Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors
◆ **2A.2C** Describe and analyze the relationship between a function and its inverse (~~quadratic and square root~~, logarithmic and exponential), including the restriction(s) on domain, which will restrict its range. ◆ **2A.5D** Solve exponential equations of the form $y = ab^x$ where a is a nonzero real number and b is greater than zero and not equal to one and ~~single~~ logarithmic equations having real solutions. ◆ **2A.2D** Use the composition of two functions, including the necessary restrictions on the domain, to determine if the functions are inverses of each other.	<u>Necessary Materials and Pre-Lesson Prep</u> ▪ SE workbook Lesson Structure: 🟥 Do Now (5 min) 🟦 INM (20 min) 🟩 Debrief (5 min) 🟪 Student Practice (20 min) 🟪 Exit Ticket (5 min) <u>Mathematical Goal of this Lesson</u> In this lesson, students will use composition to verify exp/log inverses and use Inverse Properties of Logarithms to simplify expressions. In previous grades and units' students learned how to identify the domain, range, composition of functions and inverses. In this lesson students will determine the inverses functions of logarithms and will use the properties of logarithms learned in previous lessons to simplify expressions. <u>Opportunities to CFU</u> ✓ INM #11-13 ✓ Student Practice #1-5 ✓ Debrief	Look for teachers to… ❑ Keep the focus for inverses of logarithmic functions and exponential functions on the base. ❑ Support students in the proper order of operations for logarithmic expressions. Look for students to… ❑ Find the inverse of logarithmic functions. ❑ Simplify logarithmic expressions.
Important Vocabulary ▪ domain ▪ range ▪ composition of functions ▪ logarithm ▪ base	<u>Other Notes to Inform Your Planning</u> This lesson focuses on the connection of exponential and logarithmic functions as inverses. Students will rewrite exponentials and logarithms and vice versa. During this lesson students will apply the logarithm properties as well as symmetry concepts to clarify misconceptions. **Focus on Disciplinary Literacy** Student Practice	**Student Know/Do Chart** Students can **Do** 🎯 Find the inverse of a logarithmic function. **Do** 🎯 Use the properties of logarithms to simplify logarithmic expressions. **Know** 🎓 The inverse of a logarithmic function is an exponential function.

Date: _____
Lesson 11: Applying Logarithm Properties in Any Base & Change of Base Formula

Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors

Standard(s)

◆ **2A.5C** Rewrite exponential equations as their corresponding logarithmic equations and logarithmic equations as their corresponding exponential equations.

◆ **2A.5D** Solve exponential equations of the form $y = ab^x$ where a is a nonzero real number and b is greater than zero and not equal to one and single logarithmic equations having real solutions.

Important Vocabulary

- logarithm
- base
- product property of logarithms
- quotient property of logarithms
- **inverse properties of logarithms**
- power property of logarithms
- **change of base formula**

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep
- SE workbook

Lesson Structure:
- 🟥 Do Now (5 min)
- 🟦 INM (20 min)
- 🟩 Debrief (5 min)
- 🟪 Student Practice (20 min)
- 🟪 Exit Ticket (5 min)

Mathematical Goal of this Lesson
In this lesson, students will apply logarithm properties to any base and develop and use the change of base formula.

Opportunities to CFU
- ✓ INM problems #3-6
- ✓ Student Practice #3-4
- ✓ Debrief

Other Notes to Inform Your Planning
This lesson teaches students the Change of Base Formula which allows them to then convert any logarithm to common log or natural log and approximate a value with a calculator. This is vital to being able to apply logarithms to real-world applications without the use of log tables (but we also used it heavily with log tables in the past).

Focus on Disciplinary Literacy
INM & Student Practice

Lesson Look Fors

Look for teachers to...
- ❑ Connect students to the prior lessons and how to rewrite logarithmic expressions.
- ❑ Describe for students how they can use change of base formula to evaluate logarithms using a calculator.

Look for students to...
- ❑ Apply a variety of logarithm properties to rewrite logarithmic expressions.
- ❑ Evaluate logarithmic expressions using the change of base formula.

Student Know/Do Chart

Students can

Do 🎯 Use the properties of logs and the change of base formula to simplify logarithmic expressions.

Know 📖 The Change of Base Formula allows you to change a logarithm from the given base to any other base (often used to change to the common log or natural log). $\log_b m = \frac{\log_a m}{\log_a b}$

Date: _____
Lesson 12: Solving Exponential Equations (Using Logs)

Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors
◆ **2A.5D** Solve exponential equations of the form $y = ab^x$ where a is a nonzero real number and b is greater than zero and not equal to one and ~~single~~ logarithmic equations having real solutions.	**Necessary Materials and Pre-Lesson Prep** ▪ SE workbook ▪ graphing calculator **Lesson Structure:** 🟥 Do Now (5 min) 🟦 INM (20 min) 🟩 Debrief (5 min) 🟪 Student Practice (20 min) 🟪 Exit Ticket (5 min) **Mathematical Goal of this Lesson** In this lesson, students will solve exponential equations by taking the logarithm of both sides and using inverse properties of logarithms. The students need to practice different types of problems to get familiar with the process of solving the exponential equations using logs; students will have the opportunity to explore exponential equations and understand the importance of the logarithms when solving equations. Students can use technology to calculate the logarithmic values.	**Look for teachers to…** ❑ Guide the students in selecting logs of the appropriate base to eliminate the exponential terms. **Look for students to…** ❑ Use properties of exponents to rewrite terms so that one exponential expression is equal to another then take the log of each side. ❑ Solve a variety of exponential equations by taking log of each side (often common log or natural log are best).
Important Vocabulary ▪ exponential equation ▪ exponent ▪ compound interest ▪ compound continuously interest	**Opportunities to CFU** ✓ INM problems #1 ✓ Student Practice #1-5 ✓ Debrief **Other Notes to Inform Your Planning** This lesson builds on the prior lesson in that students will again solve exponential equations, but this time they will learn to "take the log of both sides." It is important that students know to select the correct base for the logarithm they are going to use. Often students will use the common logarithm, but the natural logarithm will work in all settings. This is continued in the next lesson where students solve logarithmic equations. **Focus on Disciplinary Literacy** — INM	**Student Know/Do Chart** Students can **Do** 🎯 Solve an exponential equation using logarithms. **Know** 🎓 If the equations cannot be written as exponential expressions with the same base, then use logs on both sides of the expression, then use the concept of inverses to eliminate the one of the bases and, finally, use the properties of logarithms to solve for the variable.

Date: _____
Lesson 13: Solving Logarithmic Equations

Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors
◆ **2A.5D** Solve exponential equations of the form $y = ab^x$ where a is a nonzero real number and b is greater than zero and not equal to one and ~~single~~ logarithmic equations having real solutions. ◆ **2A.5E** Determine the reasonableness of a solution to a logarithmic equation.	**Necessary Materials and Pre-Lesson Prep** ▪ SE workbook ▪ graphing calculator **Lesson Structure:** 🟥 Do Now (5 min) 🟦 INM (20 min) 🟩 Debrief (5 min) 🟪 Student Practice (25 min) 🟪 Exit Ticket (5 min)	**Look for teachers to…** ❑ Connect students to the lessons and properties they have learned throughout the unit to solve logarithmic equations. **Look for students to…** ❑ Use properties of logarithms to simplify each side of the equations so they can solve. ❑ Solve simple logarithmic equations using the definition of a logarithm.
	Mathematical Goal of this Lesson In this lesson, students will solve logarithmic equations algebraically and identify extraneous solutions. The students need to practice different types of problems to get familiar with the process of solving logarithmic. Students can use technology to calculate the logarithmic values.	
	Opportunities to CFU ✓ INM problem #1 & 2 ✓ Student Practice #1-3 ✓ Debrief	**Student Know/Do Chart**
Important Vocabulary ▪ logarithmic equation ▪ **input/argument** ▪ base ▪ **extraneous solutions**	**Other Notes to Inform Your Planning** This lesson builds on the prior lesson and students will apply what they know about properties of logarithms to simplify and solve logarithmic equations. Students will use both, converting from a logarithm to an exponential and equality of logarithms to find missing values. It is important that students check their solutions for extraneous solutions because logarithms have limited domains. **Focus on Disciplinary Literacy** INM	Students can **Do** Solve a variety of logarithmic equations. **Know** Rearrange the terms to get an equation of the form $\log(x) = \log(y)$, then equate x and y, and then solve.

Use the following resources to review specific topics. Your exit ticket data should be used to determine individualized needs. The resources can be used in small groups, whole groups, or independent groups and be integrated with other classroom routines, like computer aligned practice and teacher-led groups.

To review or practice investigating exponential rates of change, or writing and describing exponential functions, use…

GeoGebra Interactive Applets:
- Linear vs. Exponential Rate of Change
- Quadratic vs. Exponential Rate of Change

Sample Activities & Tasks:
- TI-Calculator Activity: Comparing Linear and Exponential Data
- TI-Calculator Activity: Characteristics of Exponential Functions
- The Exponential Wheat Problem

Content Video Lessons:
- Exponential vs. Linear Growth
- Exponential vs. Linear Models: Verbal Description
- Exponential Expressions Word Problems
- Initial Value & Common Ratio of Exponential Functions
- Exponential Expressions Word Problems (Algebraic)
- Interpreting Exponential Expression Word Problem
- Exponential Growth and Decay Word Problems

To review or practice graphing and analyzing exponential functions, or transformations of exponential functions, use…

GeoGebra Interactive Applets:
- Investigating Exponential Growth and Decay Functions
- Exponential Functions and Transformations

Sample Activities & Tasks:
- TI-Calculator Activity: Graphing Exponentials
- TI-Calculator Activity: Exponential Transformations

Content Video Lessons:
- Exponential Function Graph
- Graphs of Exponential Growth
- Exponential Decay
- Transformations of Exponential Functions

To review or practice transformations of the Natural Base exponential functions, use…

GeoGebra Interactive Applets:
- What is the number e?
- Transformations of Exponential Function with Base e

Sample Activities & Tasks:
- Attributes and Transformations of Base e Functions (answers)

Content Video Lessons:
- Transformations of Base e Exponential Function with Tables
- Transformations of Exponential Functions (Base e)

To review or practice graphing and analyzing exponential data, use…

Content Video Lessons:
- Richter Scale
- What Does Magnitude Mean?
- Graphing Exponential Data on Calculator

To review or practice exponential functions to model compound interest, use…

GeoGebra Interactive Applets:
- Compound Interest

Sample Activities & Tasks:
- Compound Interest Exploration

Content Video Lessons:
- Quick Intro to Compound Interest
- Introduction to Compound Interest
- Compound Interest Example-Finding Principal

To review or practice using exponential regression to model real data, use…

GeoGebra Interactive Applets:
- Exponential Growth of a Population
- Exponential Regression Fit

Desmos Interactive Applets:
- Linear vs. Exponential Regression

Sample Activities & Tasks:
- Exponential Regression Model Example (Using TI-Nspire)
- Choosing the Best Model Lab: Exponential Regression

Content Video Lessons:
- Exponential Regression on TI-Nspire Calculator
- Exponential Regression Example

Date: _____
Unit 6 Exam

Standard(s)	Notes for Intellectual Preparation & Lesson Planning
◆ **2A.2A** Graph the functions ~~$f(x) = \sqrt{x}$, $f(x) = 1/x$, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$~~, $f(x) = b^x$, ~~$f(x) = |x|$~~, and $f(x) = \log_b(x)$ where b is 2, 10, and e, and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval. **Problem #3** ◆ **2A.2C** Describe and analyze the relationship between a function and its inverse (~~quadratic and square root~~, logarithmic and exponential), including the restriction(s) on domain, which will restrict its range. **Problem #3** ◆ **2A.5A** Determine the effects on the key attributes on the graphs of $f(x) = b^x$ and $f(x) = \log_b(x)$ where b is 2, 10, and e when $f(x)$ is replaced by $af(x)$, $f(x) + d$, and $f(x - c)$ for specific positive and negative values of a, c, and d. **Problems #2, 8** ◆ **2A.5D** Solve exponential equations of the form $y = ab^x$ where a is a nonzero real number and b is greater than zero and not equal to one and single logarithmic equations having real solutions. **Problems #4, 6, 9abc** ◆ **2A.8C** Predict and make decisions and critical judgments from a given set of data using linear, quadratic, and exponential models. **Problem #11bc** ◆ **2A.2D** Use the composition of two functions, including the necessary restrictions on the domain, to determine if the functions are inverses of each other. **Problem #5** ◆ **2A.5B** Formulate exponential and logarithmic equations that model real-world situations, including exponential relationships written in recursive notation. **Problem #10** ◆ **2A.5C** Rewrite exponential equations as their corresponding logarithmic equations and logarithmic equations as their corresponding exponential equations. **Problem #7** ◆ **2A.5E** Determine the reasonableness of a solution to a logarithmic equation. **Problem #9c** ◆ **2A.8A** Analyze data to select the appropriate model from among linear, quadratic, and exponential models. **Problems #1, 11a**	<u>Necessary Materials and Pre-Lesson Prep</u> ▪ Algebra 2 Unit 6 Exam ▪ Assessment Companion for Algebra 2 Unit 6 Exam found on Curriculum Corner <u>Notes to Inform Your Planning</u> Review the Unit 6 Exam on Curriculum Corner. Internalize & create an exemplar for the assessment prior to teaching the unit as part of unpacking the unit. Use the exemplar to spar with the answer key provided on Curriculum Corner. Administer the Unit 6 Exam following guidance on the Scope & Sequence to ensure the scoring deadline is met.

UNPACKED STANDARDS

Focus standards for this unit.

Standards Clarification		
Standards	**Specificity**	**Notes/Explanations/Examples**

2A.2A Graph the functions $f(x) = \sqrt{x}$, $f(x) = 1/x$, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = |x|$, and $f(x) = \log_b(x)$ where b is 2, 10, and e, and, when applicable, analyze the key attributes such as domain, range, intercepts, ~~symmetries~~, asymptotic behavior~~, and maximum and minimum given an interval.~~	Graph: Including, but not limited to:
	• Representations of functions, including graphs, tables, and algebraic generalizations
	  o Exponential, $f(x) = b^x$, where b is 2, 10, and e
	  o Logarithmic, $f(x) = \log_b(x)$, where b is 2, 10, and e
	• Connections between representations of families of functions
	• Comparison of similarities and differences of families of functions
	Analyze: Including, but not limited to:
	• Domain and range of the function
	  o Domain – set of input values for the independent variable over which the function is defined
	  o Range – set of output values for the dependent variable over which the function is defined
	  o Representation for domain and range
	  o Domain and range of the function versus domain and range of the contextual situation
	• Key attributes of functions
	  o Intercepts/Zeros
	  o Asymptotic behavior – behavior such that as x approaches infinity, $f(x)$ approaches a given value
	• Use key attributes to recognize and sketch graphs
	• Application of key attributes to real-world problem situations
		• Grade Level(s):
		  o The notation $\Re$ represents the set of real numbers, and the notation Z represents the set of integers.
		  o Algebra I studied parent functions $f(x) = x$, $f(x) = x^2$, and $f(x) = b^x$ and their key attributes.
		  o Precalculus will study polynomial, power, trigonometric, inverse trigonometric, and piecewise defined functions, including step functions.
		  o Various mathematical process standards will be applied to this student expectation as appropriate.

2A.2C Describe and analyze the relationship between a function and its inverse (~~quadratic and square root,~~ logarithmic and exponential), including the restriction(s) on domain, which will restrict its range.	Including, but not limited to:
	• Relationships between functions and their inverses
	  o All inverses of functions are relations.
	  o Inverses of one-to-one functions are functions.
	  o Inverses of functions that are not one-to-one can be made functions by restricting the domain of the original function, $f(x)$.
	• Characteristics of inverse relations
	  o Interchange of independent (x) and dependent (y) coordinates in ordered pairs
	  o Reflection over $y = x$
	• Domain and range of the function versus domain and range of the inverse of the given function
	• Functionality of the inverse of the given function tabularly and graphically
	  o Exponential function and logarithmic function, $f(x) = b^x$ and $g(x) = \log_b(x)$ where b is 2, 10, and e
		• Grade Level(s):
		  o Algebra I determined if relations represented a function.
		  o Algebra II introduces inverse of a function and restricting domain to maintain functionality.
		  o Various mathematical process standards will be applied to this student expectation as appropriate.

Standards Clarification		
Standards	Specificity	Notes/Explanations/Examples

2A.5A Determine the effects on the key attributes on the graphs of $f(x) = b^x$ and $f(x) = \log_b(x)$ where b is 2, 10, and e when $f(x)$ is replaced by $af(x)$, $f(x) + d$, and $f(x - c)$ for specific positive and negative values of a, c, and d.	Including, but not limited to:
	• General form of the power function
	  o Exponential functions, $f(x) = b^x$, where b is 2, 10, and e
	  o Logarithmic functions, $y = \log_b(x)$, where b is 2, 10, and e
	• Representations with and without technology
	  o Graphs
	  o Tables
	  o Verbal descriptions
	  o Algebraic generalizations
	• Key attributes
	  o Intercepts
	  o Asymptotes
	• Effects on the graphs of $f(x) = b^x$ and $y = \log_b(x)$ when parameters a, b, c, and d are changed in $f(x) = a \cdot b^{(x-c)} + d$ and $f(x) = a \cdot \log_b(x - c) + d$,
	  o Effects on the graphs of $f(x) = 2^x$ and $f(x) = \log_2(x)$ when $f(x)$ is replaced by $af(x)$ with and without technology
	  o Effects on the graphs of $f(x) = 10^x$ and $f(x) = \log(x)$, when $f(x)$ is replaced by $f(x - c)$ with and without technology
	  o Effects on the graphs of $f(x) = e^x$ and $f(x) = \ln(x)$, when $f(x)$ is replaced by $f(x) + d$ with and without technology
	Connections between the critical attributes of transformed functions and $f(x) = b^x$ and $y = \log_b(x)$
	  o Determination of parameter changes given a graphical or algebraic representation
	  o Determination of a graphical representation given the algebraic representation or parameter changes
	  o Determination of an algebraic representation given the graphical representation or parameter changes
	Descriptions of the effects on the domain and range by the parameter changes
	Effects of multiple parameter changes
	  o Mathematical problem situation
	  o Effects of parameter changes in real-world problem situations
		• Grade Level(s):
		  o Algebra I determined effects on the graphs of the parent functions, $f(x) = x$ and $f(x) = x^2$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a, b, c, and d.
		  o Algebra II continues to investigate the exponential parent function and introduces logarithmic parent function and transformations of both functions.
		  o Various mathematical process standards will be applied to this student expectation as appropriate.

2A.5D Solve exponential equations of the form $y = ab^x$ where a is a nonzero real number and b is greater than zero and not equal to one and single logarithmic equations having real solutions.	Including, but not limited to:
	• Exponential equation, $y = ab^x$
	  o a – initial value at $x = 0$
	  o b – common ratio
	• Solving exponential equations
	  o Application of laws (properties) of exponents
	  o Application of logarithms as necessary
	• Real-world problem situations modeled by exponential functions
	  o Exponential growth
	  o Exponential decay
	• Single logarithmic equation, $y = \log_b(x)$
		• Grade Level(s):
		  o Algebra I applied exponential functions to problem situations using tables, graphs, and the algebraic generalization, $f(x) = a \cdot b^x$.
		  o Algebra II solves exponential equations algebraically.
		  o Algebra II introduces logarithms and solving logarithmic equations.
		  o Precalculus will use properties of logarithms to solve equations.

Standards Clarification		
Standards	**Specificity**	**Notes/Explanations/Examples**

	• x – argument
	• b – base
	• y – exponent
	  • Solving logarithmic equations
	    o Transformation to exponential form as necessary
	  • Real-world problem situations modeled by logarithmic functions
		o Various mathematical process standards will be applied to this student expectation as appropriate.

2A.8C Predict and make decisions and critical judgments from a given set of data using linear, quadratic, and exponential models.	Including, but not limited to:
	• Mathematical and real-world problem situations modeled by linear, quadratic, and exponential functions and equations
	• Predictions, decisions, and critical judgments from function models
	• Justification of reasonableness of solutions in terms of mathematical and real-world problem situations
	  o Mathematical justification
		• Grade Level(s):
		  o Algebra I introduced the linear, quadratic, and exponential functions.
		  o Algebra I introduced the correlation coefficient as a measure of the strength of linear association.
		  o Algebra I applied linear, quadratic, and exponential functions to model and make predictions in real-world problem situations.
		  o Various mathematical process standards will be applied to this student expectation as appropriate.

2A.2D Use the composition of two functions, including the necessary restrictions on the domain, to determine if the functions are inverses of each other.	Use: Including, but not limited to:
	• Composition of functions – process of substituting one function into another function to create a new function such that the range of one function becomes the domain of the other function
	• Composition notation given $f(x)$ and $g(x)$
	  o Verbal
	  o Symbolic
	To Determine: Including, but not limited to:
	• Characteristics of inverse relations
	  o Interchange of independent (x) and dependent (y) coordinates in ordered pairs
	  o Interchange of independent (x) and dependent (y) coordinates in an equation and resolving for y
	  o Reflection over $y = x$
	  o Domain of the function becomes an appropriate range of the inverse function
	  o Range of the function becomes an appropriate domain of the inverse function
	  o Composed as $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$
	• Domain and range of the function versus domain and range of the inverse of the given function
		• Grade Level(s):
		  o Algebra II introduces inverse of a function.
		  o Various mathematical process standards will be applied to this student expectation as appropriate.

Standards Clarification

Standards	Specificity	Notes/Explanations/Examples

2A.5B Formulate exponential and logarithmic equations that model real-world situations, including exponential relationships written in recursive notation.	Including, but not limited to:
	• Data collection activities with and without technology
	  o Data modeled by exponential functions
	  o Data modeled by logarithmic functions
	• Real-world problem situations
	  o Real-world problem situations modeled by exponential functions
	  o Real-world problem situations modeled by logarithmic functions
	• Representations of exponential and logarithmic equations
	  o Tables/graphs
	  o Verbal descriptions
	• Technology methods
	  o Transformations of $f(x) = b^x$ and $y = \log_b(x)$
	  o Exponential regression
	  o Logarithmic regression
		• Grade Level(s):
		  o Algebra II introduces formulating exponential and logarithmic equations.
		  o Various mathematical process standards will be applied to this student expectation as appropriate.

2A.5C Rewrite exponential equations as their corresponding logarithmic equations and logarithmic equations as their corresponding exponential equations.	Including, but not limited to:
	• Laws (properties) of exponents
	  o Product of powers (multiplication when bases are the same): $a^m \cdot a = a^{m+n}$
	  o Quotient of powers (division when bases the same): $\frac{a^m}{a^n} = a^{m-n}$
	  o Power to a power: $(a^m)^n = a^{mn}$
	  o Negative exponents: $a^{-n} = \frac{1}{a^n}$
	  o Zero exponent: $a^0 = 1$
	• Laws (properties) of logarithms
	  o Product: $\log_b(p \cdot q) = \log_b(p) + \log_b(q)$; $\ln(p \cdot q) = \ln(p) + \ln(q)$
	  o Quotient: $\log_b\frac{p}{q} = \log_b(p) - \log_b(q)$; $\ln\frac{p}{q} = \ln(p) - \ln(q)$
	  o Power: $\log(p)^q = q$ bull; $\log_b(p)$; $\ln(p)^q = q \cdot \ln(p)$
	  o Reciprocal: $\log_b\frac{1}{p} = -\log_b(p)$; $\ln\frac{1}{p} = -\ln(p)$
	  o Log of base: $\log_b(b) = 1$; $\ln(e) = 1$
	  o Log of 1: $\log_b(1) = 0$; $\ln(1) = 0$
	• Exponential equations to corresponding logarithmic equations
	  o $b^x = A \rightarrow \log_b(A) = x$
	• Logarithmic equations to corresponding exponential equations
	  o $\log_b(A) = x \rightarrow b^x = A$
		• Grade Level(s):
		  o Algebra I applied exponential functions to problem situations using tables, graphs, and the algebraic generalization, $f(x) = a \cdot b^x$.
		  o Algebra II introduces logarithms.
		  o Algebra II connects between exponential equations and logarithmic equations.
		  o Various mathematical process standards will be applied to this student expectation as appropriate.

2A.5E Determine the reasonableness of a solution to a logarithmic equation.	Including, but not limited to:
	• Justification of solutions to logarithmic equations with and without technology
	  o Verbal description, Tables, Graphs
	  o Substitution of solutions into original functions
	• Justification of reasonableness of solutions in terms of mathematical and real-world problem situations or data collections
		• Grade Level(s):
		  o Algebra II introduces logarithms and solving logarithmic equations.
		  o Various mathematical process standards will be applied to this student expectation as appropriate.

Standards Clarification

Standards	Specificity	Notes/Explanations/Examples

2A.8A Analyze data to select the appropriate model from ~~among linear, quadratic, and~~ exponential models.	Analyze: Including, but not limited to:
	• Data collected from data collection devices
	• Data given in mathematical problem situations
	• Types of data
	   o Exponential
	To Select: Including, but not limited to:
	• Data collected from data collection devices
	• Data given in real-world problem situations
	• Data relationships
	   o Exponential
	• Data representations
	   o Data tables
	   o Graphs/scatterplots
	   o Verbal descriptions
	   o Algebraic generalizations
		• Grade Level(s):
		   o Algebra I introduced the linear, quadratic, and exponential functions.
		   o Algebra II expands on transformations and applications of exponential functions.
		   o Various mathematical process standards will be applied to this student expectation as appropriate.

VERTICAL STANDARDS

This section details the **progression** of key student expectations (SEs) in the courses **before** and **after** this course. This will help you understand what **prior knowledge skills to build upon** and guide you in knowing what **skills you are preparing your students** for in the subsequent course.

Algebra 1	Algebra 2	Pre-Calculus
• **A.9A** Determine the domain and range of exponential functions of the form $f(x) = ab^x$ and represent the domain and range using inequalities. • **A.9B** Interpret the meaning of the values of a and b in exponential functions of the form $f(x) = ab^x$ in real-world problems. • **A.9C** Write exponential functions in the form $f(x) = ab^x$ (where b is a rational number) to describe problems arising from mathematical and real-world situations, including growth and decay. • **A.9D** Graph exponential functions that model growth and decay and identify key features, including y-intercept and asymptote, in mathematical and real-world problems. • **A.9E** Write, using technology, exponential functions that provide a reasonable fit to data and make predictions for real-world problems.	• **2A.2A** Graph the functions $f(x) = b^x$ and $f(x) = \log_b(x)$ where b is 2, 10, and e, and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval. • **2A.2C** Describe and analyze the relationship between a function and its inverse (logarithmic and exponential), including the restriction(s) on domain, which will restrict its range. • **2A.2D** Use the composition of two functions, including the necessary restrictions on the domain, to determine if the functions are inverses of each other. • **2A.5A** Determine the effects on the key attributes on the graphs of $f(x) = b^x$ and $f(x) = \log_b(x)$ where b is 2, 10, and e when $f(x)$ is replaced by $af(x)$, $f(x) + d$, and $f(x - c)$ for specific positive and negative values of a, c, d. • **2A.5B** Formulate exponential and logarithmic equations that model real-world situations, including exponential relationships written in recursive notation. • **2A.5C** Rewrite exponential equations as their corresponding logarithmic equations and logarithmic equations as their corresponding exponential equations. • **2A.5D** Solve exponential equations of the form $y = ab^x$ where a is a nonzero real number and b is greater than zero and not equal to one and single logarithmic equations having real solutions. • **2A.5E** Determine the reasonableness of a solution to a logarithmic equation. • **2A.8A** Analyze data to select the appropriate model from among linear, quadratic, and exponential models. • **2A.8B** Use regression methods available through technology to write an exponential function from a given set of data. • **2A.8C** Predict and make decisions and critical judgments from a given set of data using linear, quadratic, and exponential models.	• **(2.G)** Graph functions, including exponential, logarithmic, ~~sine, cosine, rational, polynomial, and power functions~~ and their transformations, including $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of $a, b, c,$ and d, in mathematical and real-world problems. • **(2.I)** Determine and analyze the key features of exponential, logarithmic, ~~rational, polynomial, power, trigonometric, inverse trigonometric, and piecewise defined functions, including step functions~~ such as domain, range, symmetry, relative maximum/minimum, zeros, asymptotes, and intervals over which the function is increasing/decreasing. • **(2.J)** Analyze and describe end behavior of functions, including exponential, logarithmic, ~~rational, polynomial, and power functions,~~ using infinity notation to communicate this characteristic in mathematical and real-world problems. • **(2.N)** Analyze situations modeled by functions, including exponential, logarithmic, ~~rational, polynomial, and power functions~~, to solve real-world problems. • **(5.G)** Use the properties of logarithms to evaluate or transform logarithmic expressions. • **(5.H)** Generate and solve logarithmic equations in mathematical and real-world problems. • **(5.I)** Generate and solve exponential equations in mathematical and real-world problems.